RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR
B.A./B.SC. SECOND SEMESTER (January – June), 2012
Mid-Semester Examination, March 2012

Date : 22/03/2012   **MATHEMATICS (General)**
Time : 11 am – 12 noon   Paper : II   Full Marks : 25

1. Answer **any one** question : [1×5]

   a) Prove that the equation to the straight lines through the origin each of which makes and angle α with the straight line $y = x$ is $x^2 - 2xy \sec 2\alpha + y^2 = 0$. [5]

   b) Find the angle of rotation of the co-ordinate axes about the origin which will transform the equation $x^2 - y^2 = 4$ to $x'y' = 2$.

   Find the nature of the given equation $11x^2 + 4xy + 14y^2 + 26x + 32y + 23 = 0$. [3+2]

2. Answer **any one** question : [1×4]

   a) If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ be the angle between them, then show that $\sin \frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$. [4]

   b) Prove that $\left|\vec{a} \times \vec{b}\right|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$. [4]

3. Answer **any two** questions : [2×4]

   a) Prove that a sequence can have at most one limit. [4]

   b) Prove that $\lim_{n \to \infty} \left(\frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + ... + \frac{1}{\sqrt{n^2 + n}} \right) = 1$. [4]

   c) Prove that the series $\frac{1}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \frac{3}{5 \cdot 7} + ...$ is divergent. [4]

4. Answer **any one** question : [4]

   a) Find the value of $\int_{0}^{1} \frac{\log(1 + x)}{1 + x^2} dx$.

   b) Evaluate $\int_{0}^{\pi/2} \sin^n x \, dx$, where n is an even positive integer greater than 1.

5. Answer **any one** question : [4]

   a) Solve the differential equation : $(2x - y + 1)dx = (6x - 5y + 4)dy$
   b) Solve the differential equation : $(y^2 e^x + 2xy)dx - x^2 dy = 0$

ॐ ✺ ॐ